LIMB DARKENING

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Abstract

Resolved stars (for example the Sun) are observed to be darker at the edge, that is the limb, than at the centre of the disk. This is called limb darkening. Explanation of limb darkening by solving the radiative transfer equation is discussed in this essay. The applications of the study of limb darkening is also discussed.

Introduction

When we observe the sun in a reduced intensity, the edge(which is called the limb) of the sun appears to be darker than the centre of the disk. The decrease in intensity at visible wavelengths from the centre of the sun to the limb of the sun's disk is limb darkening.

The reason why the limb appears darker for the stars compared to that of the centre of their disks is that the stars have a temperature gradient from the core to the surface. They are hotter in the deeper parts that in the outer parts of the atmosphere. The temperature of the interior of the sun is about 40 million degree celsius and that of the photosphere is about 10000 to 20000 degree celsius.

Though the photosphere has a very small thickness about few hundred kilometers, it has a large temperature gradient as we go deepeer into it. The deep photosphere has a considerably higher temperature that at its top layer.

The light from the central portion of the disk is radiated radially towards us, whereas the light from closer to the limb has to pass through a greater thickness of the solar atmosphere.

For a given depth of penetration, we can see deeper into the photosphere at the center of the disk than at the limb, where our line of sight passes through the cooler gases. The maximum depth to which the human eyes can probe into the photosphere is the optical depth given by $\tau = 2/3$
Our line of sight probes more deeply into the hotter parts at the centre of the disk. But at the limb the line of sight enters the photosphere at a large slant angle and

does not probe deep into hotter regions, and hence we see only the cooler regions. This causes the sun to show a much pronounced darkening at the limb.

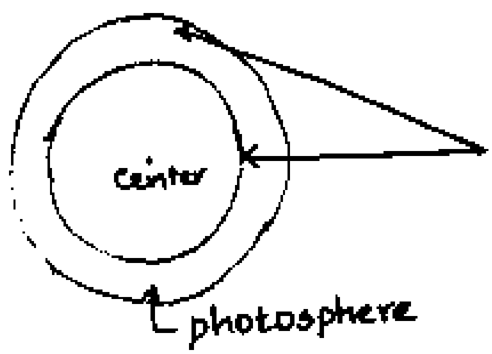


Fig 1: This figure shows the depth upto which our eyes probe into the photosphere of the sun at the center and at the limb.

Temperature profile at the photosphere

The disk center brightness corresponds to a temperature of about 6390 K, characteristic of the photosphere, while the brightness near the limb corresponds to the temperature of about 5000 K.

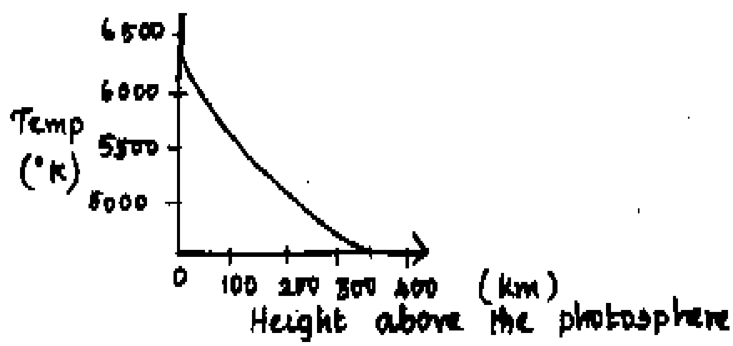


Fig 2: Temperature profile of the photosphere
The temperature decreases as we reach the surface of the photosphere from the bottom layer of it.

The evidence of the absorption lines can also be interpreted to conclude that the atmosphere of the sun is characterised by cooler rarified regions which overlie the hotter dense regions.

Explanation of limb darkening as a measure of brightness temperature and color temperature of the solar surface

By measuring the brightness (or the specific intensity) of the solar image, we can detemine the solar surface temperature, by comparing it to the intensity of a black-body at the particular frequency ν. This gives the brightness temperature.

The measurement of relative intensity of solar emission at different colors give a color temperature in agreement with the brightness temperature.

Now looking at the brightness temperature of the solar image we find that it is 50 % brighter at the center than at the limb. This implies that the brightness temperature is 10 % higher at the center than at the limb.

Therefore limb darkening represents a decrease in the brightness temperature towards the limb and hence we expect the color temperature also to decrease towards the limb. That is the limb should appear redder in color that the center of the disk. Detailed measurments show that indeed the emission is noticeably redder in color towards the limb of the sun.

The upshot of both brightness and color temperature measurements is that

- the sun's surface temperature (while equalling 5800 K when averaged over the disk) is significantly hotter about 6390 K at the center of the disk.
- and it is significantly cooler about 5000 K near the limb.

Explanation of limb darkening by the solution of Radiative transfer equation

The radiative transfer equation is given as

$$\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu \tag{1}$$

where

$$d\tau_\nu = -\alpha_\nu ds \tag{2}$$

as τ_ν is measured in the opposite direction. ds is the distance with respect to an arbitrary orientation θ of the radiation.

Therefore for radiation in the radial direction

$$ds = dr\cos\theta \tag{3}$$

and therefore

$$\frac{dI_\nu}{d\tau_\nu}\cos\theta = I_\nu - S_\nu \tag{4}$$

Let $\mu = \cos\,\theta$
Therefore the radiative transfer equation is

$$\mu\frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - S(\tau) \tag{5}$$

Considering the radiative equilibrium and isotropic scattering,

$$j_\nu = \alpha_\nu j_\nu \tag{6}$$

where J_ν is the mean intensity.
Therefore

$$s(\tau) = J(\tau) = \frac{1}{4\pi}\int I(\tau,\mu)d\Omega \tag{7}$$

Equation (5) can be written as

$$\mu\frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - \frac{1}{2}\int_{-1}^{1} I(\tau,\mu)d\mu \tag{8}$$

As a solution to this equation, the assumption of near isotropy is introduced by considering that intensity is a power series in μ, only with terms upto linear,

$$I(\tau,\mu) = I_0 + I_1\mu \tag{9}$$

where I_0 is isotropic intensity and I_1 is an anisotropic component.

Now the mean intensity is

$$J(\tau) = \frac{1}{2}\int_{-1}^{1}(I_0 + I_1\mu)d\mu = I_0 \tag{10}$$

That is the mean intensity is isotropic intensity.

The net flux is given by

$$F(\tau) = \int I(\tau, \mu) \cos\theta d\Omega \tag{11}$$

That is

$$F(\tau) = \frac{4}{3}\pi I_1 \tag{12}$$

Now the net flux is the same for all τ, and I_1 also should be independent of τ.

Substituting equations (9) and (10) in equation (8) which is the transfer equation and simplyfying we get

$$\mu\frac{dI_0}{d\tau} = I_1\mu \tag{13}$$

Integrating we get,

$$I_0 = I_1\tau + c \tag{14}$$

Substituting equation (14) in (9) we get,

$$I(\tau, \mu) = I_0 + I_1\mu = I_1(\tau + \mu) + c \tag{15}$$

Considering the net inward flux equal to zero,

$$F^{-}(\tau) = -\int_0^{2\pi}\int_{\pi/2}^{\pi} I(\theta, \tau) \cos\theta d\Omega \tag{16}$$

Evaluating this we get the value of c as

$$c = \frac{2}{3}I_1 \tag{17}$$

We have from equation (12)
F(τ) = $\frac{4}{3}$ π I_1

Therefore I_1 = 3 F(0)/(4π)

Substituting this and equation (17) in equation (15) we get,

$$I(\tau, \mu) = \frac{3}{4\pi}(\tau + \mu + \frac{2}{3})F(0) \tag{18}$$

This equation shows that the intensity at the depth τ with radiation at θ is given in terms of observable net flux at the surface.

Equation (18) gives the intensity at the optical depth τ

At the surface of the star, optical depth $\tau = 0$
Therefore the intensity at the surface is given by

$$I(0,\mu) = \frac{3}{4\pi}(\mu + \frac{2}{3})F(0) \tag{19}$$

$$I(0,\mu) = \frac{3}{4\pi}(\cos\theta + \frac{2}{3})F(0) \tag{20}$$

Limb darkening can be defined as the ratio of intensity along θ to that of the center of the disk where θ=0
Therefore

$$\frac{I(0,\theta)}{I(0,0)} = \frac{\frac{3}{4\pi}(\cos\theta + \frac{2}{3})F(0)}{\frac{3}{4\pi}(1 + \frac{2}{3})F(0)} \tag{21}$$

$$= \frac{3}{5}(\cos\theta + \frac{2}{3}) \tag{22}$$

That is

$$\frac{I(0,\theta)}{I(0,0)} = 0.4 + 0.6\cos\theta \tag{23}$$

This equation shows that the intensity along θ is $(0.4 + 0.6 \cos\theta)$ times the intensity at the center of the disk.

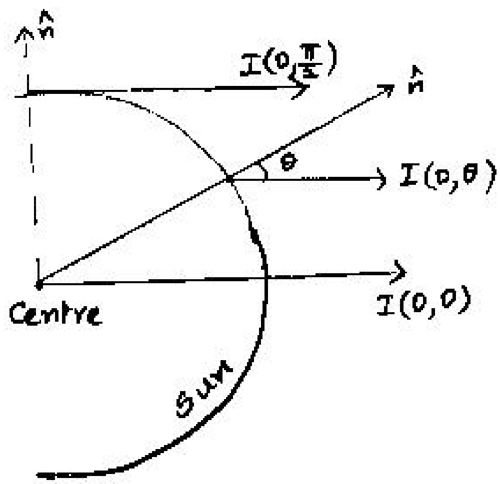


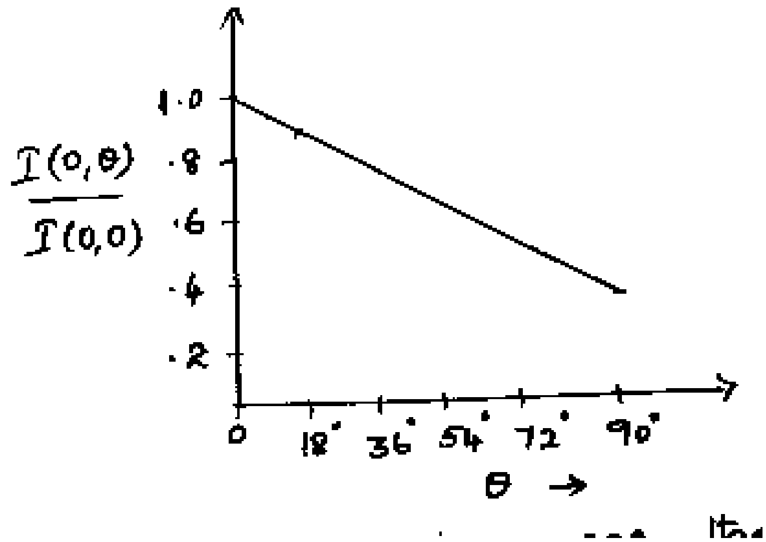
Fig 3: This figure shows that as θ increases the intensity decreases. Therefore for $\theta = 0$, at the center of the disk, the intensity is maximum.

And for $\theta = \pi/2$, at the limb, we get

$$I(0, \pi/2) = 0.4 I(0, 0) \tag{24}$$

From this equation it is clear that the intensity at the limb is 40% of the intensity at the center of the disk.

The plot of I(0,θ)/I(0,0), ie. the ratio of intensities vs the angle θ is shown below.



Thus as θ increases, the intensity decreases.

From the equation, it is clear that the intensity at the limb is 40% of that at the center and this agrees with measurements of the intensity at the solar disk.

The equation can also be written as

$$\frac{I(0,\theta)}{I(0,0)} = 1 - 0.6 + 0.6\cos\theta \tag{25}$$

$$= 1 - u + u\cos\theta$$

$$= 1 - u\,(1 - \cos\theta)$$ That is

$$\frac{I(0,\theta)}{I(0,0)} = 1 - u(1-\mu) \tag{26}$$

Where u=0.6 is known as the limb darkening coefficient.

Why do we have to study Limb Darkening?

•Measurement of a star's apparent diameter with an intensity interferometer yields its geometrical radius and its effective temperature when one knows or measures the specific center to limb-darkening.

- Knowledge of limb darkening effect from the center of the disk of the star to its limb, ie. the variation in the specific intensity accross the stellar disc is important in the determination of the angular diameter from the observed fluxes or from interferometric methods.

- Limb darkening observations and absolute radiation measurements are fundamental parameters for the construction and verification of solar model atmosphers.

- By studying limb darkening at a variety of wavelengths we may hope to understand something about the atmospheric chemical composition and structure.

- Studies on Limb darkening were once restricted to very few stars, but now it is broadly possible with the advent of optical interferometers.

- Ground-based interferometry has finally reached a stage in which accurated determination of Cepheid diameters is feasible. Determining these diameters is the base for calibrating the period-luminosity relation for Classical Cepheids and thus the Extragalactic distance scale.